

## TIME SERIES MODELS WITH GENERALIZED GEOMETRIC LINNIK MARGINALS

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### ABSTRACT

Time series models with non-Gaussian marginal distributions have received much attention in recent years. This include autoregressive models with exponential, Pareto, Weibull, logistic, mixed Gamma, Laplace, Cauchy and Stable marginal distributions. In this paper, autoregressive models with Type I and Type II generalized Geometric Linnik marginal are developed.

### KEYWORDS

Geometric Linnik Distribution, Generalized Geometric Linnik Distribution, Autoregressive Models

## 1. Introduction

The analysis of time series in the classical set up is based on the assumption that the observed series is a realization from a Gaussian sequence. However, there are many situations where the naturally occurring data show a tendency to follow heavy tailed distributions that cannot be modeled by a Gaussian distribution. The usual technique of transferring data to use a Gaussian model also fails in certain situations (see, [4]). Hence a number of non-Gaussian autoregressive models have been introduced by various researchers (see, [2] and [3]).

The study of non-Gaussian autoregressive models began with the pioneering work of [1]. They have considered an AR(1) model with exponential( $\mu$ ) marginal distribution. The model is given by  $x_0 = \varepsilon_1$  and for  $n = 1, 2, \dots$

$$X_n = \rho X_{n-1} + \begin{cases} 0 & w \cdot p. & p \\ \varepsilon_n & w \cdot p. & (1-p) \end{cases} \quad (1)$$

and w.p. stands for with probability,  $0 \leq \rho \leq 1$  and  $\{\varepsilon_n\}$  is a sequence of independent and identically distributed exponential random variables.

[5] developed Pakes generalized Linnik first order autoregressive process and studied its properties. [6] studied some properties of type I and type II Generalized Geometric Linnik Distribution. In this paper we develop autoregressive models with Geometric Linnik marginal distribution.

## 2. TIME SERIES MODELS WITH GEOMETRIC LINNIK MARGINALS

**Definition 2.1.** A random variable  $X$  on  $R$  is said to have geometric Linnik distribution and write  $X \stackrel{d}{=} GL(\alpha, \lambda)$  if its characteristic function  $\phi(t)$  is

$$\phi(t) = \frac{1}{1 + \ln(1 + \lambda|t|^\alpha)}, t \in R, 0 < \alpha \leq 2, \lambda > 0 \quad (2)$$

**Definition 2.2.** A random variable  $X$  on  $R$  is said to have type I generalized geometric Linnik distribution and write  $X \stackrel{d}{=} GeGL_1(\alpha, \lambda, p)$  if it has the characteristic function

$$\phi(t) = \frac{1}{1 + p \ln(1 + \lambda|t|^\alpha)}, p > 0, \lambda > 0, 0 < \alpha \leq 2 \quad (3)$$

**Definition 2.3.** A random variable  $X$  on  $R$  is said to have type II Generalized Geometric Linnik distribution and write  $X \stackrel{d}{=} GeGL_2(\alpha, \lambda, \tau)$  if it has the characteristic function

$$\phi(t) = \left[ \frac{1}{1 + \ln(1 + \lambda|t|^\alpha)} \right]^\tau, t \in R, \lambda, \tau > 0, 0 < \alpha \leq 2. \quad (4)$$

Note that when  $\tau = 1$ , type II Generalized Geometric Linnik distribution reduces to geometric Linnik distribution.

**Theorem 2.4.** Let  $\{X_n, n \geq 1\}$  be defined as

$$X_n = \begin{cases} \varepsilon_n & w \cdot p. \quad p \\ X_{n-1} + \varepsilon_n & w \cdot p. \quad (1 - p) \end{cases} \quad (5)$$

where  $\{\varepsilon_n\}$  is a sequence of independent and identically distributed random variables. A necessary and sufficient condition that  $\{X_n\}$  is strictly stationary Markov process with  $GL(\alpha, \lambda)$  marginals is that  $\{\varepsilon_n\}$  are distributed as  $GeGL_1(\alpha, \lambda, p)$ .

**Proof.** Taking characteristic functions on both sides of (5), we get

$$\phi_{X_n}(t) = p\phi_{\varepsilon_n}(t) + (1 - p)\phi_{X_{n-1}}(t)\phi_{\varepsilon_n}(t)$$

If  $\{X_n\}$  is stationary, then  $\phi_X(t) = p\phi_\varepsilon(t) + (1 - p)\phi_X(t)\phi_\varepsilon(t)$ .  
That is,

$$\phi_\varepsilon(t) = \frac{\phi_X(t)}{p + (1 - p)\phi_X(t)}$$

If  $\phi_X(t) = \frac{1}{1+\ln(1+\lambda|t|^\alpha)}$ , then  $\phi_\varepsilon(t) = \frac{1}{1+p\ln(1+\lambda|t|^\alpha)}$ .

Conversely, if  $\{\varepsilon_n\}$  are independent and identically distributed as  $GeGL_1(\alpha, \lambda, p)$ , then

$$\begin{aligned}\phi_{X_1}(t) &= p \frac{1}{1+p\ln(1+\lambda|t|^\alpha)} + (1-p) \frac{1}{1+\ln(1+\lambda|t|^\alpha)} \frac{1}{1+p\ln(1+\lambda|t|^\alpha)} \\ &= \frac{1}{1+p\ln(1+\lambda|t|^\alpha)} \left[ \frac{p+p\ln(1+\lambda|t|^\alpha)+1-p}{1+\ln(1+\lambda|t|^\alpha)} \right] = \frac{1}{1+\ln(1+\lambda|t|^\alpha)}\end{aligned}$$

. If  $X_{n-1} \stackrel{d}{=} GL(\alpha, \lambda)$  then we get  $X_n \stackrel{d}{=} GL(\alpha, \lambda)$ . Hence the process  $\{X_n\}$  is strictly stationary. This completes the proof.  $\square$

Consider the  $k^{th}$  order autoregressive process

$$X_n = \begin{cases} \varepsilon_n & w \cdot p & p \\ X_{n-1} + \varepsilon_n & w \cdot p & p_1 \\ X_{n-2} + \varepsilon_n & w \cdot p & p_2 \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ X_{n-k} + \varepsilon_n & w \cdot p & p_k \end{cases} \quad (6)$$

where  $p + p_1 + p_2 + \dots + p_k = 1$ ,  $0 < p_i < 1, i = 1, 2, \dots, k$  and  $\{\varepsilon_n\}$  is a sequence of independent and identically distributed random variables independent of  $X_{n-1}, X_{n-2}, \dots$

Taking characteristic functions on both sides of (6), we get

$$\phi_{X_n}(t) = p\phi_{\varepsilon_n}(t) + p_1\phi_{X_{n-1}}(t)\phi_{\varepsilon_n}(t) + p_2\phi_{X_{n-2}}(t)\phi_{\varepsilon_n}(t) + \dots + p_k\phi_{X_{n-k}}(t)\phi_{\varepsilon_n}(t).$$

That is,

$$\phi_{\varepsilon_n}(t) = \frac{\phi_X(t)}{p + (1-p)\phi_X(t)}.$$

Following similar lines in Theorem 2.4., we get the following result.

**Theorem 2.5.** A necessary and sufficient condition that the model (6) defines AR(k) process with  $GL(\alpha, \lambda)$  distribution is that  $\{\varepsilon_n\}$  is distributed as  $GeGL_1(\alpha, \lambda, p)$ .

The model developed in this paper can be used for modeling stock price returns, speech waves etc, as an alternative to Generalized Linnik laws and Pakes generalized Linnik laws.

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